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Sets

"The Real ~~analysis~~ number system":

Set: A set is a well defined collection of distinct objects
→ By well defined we mean there is no confusion regarding inclusion or exclusion of objects. Sets are usually denoted with capital letters and the member of the set are denoted by small letters.

"Following notations will be used for some of the specific sets that are commonly used"

$\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ = the set of all natural no's.
$W = \{0, 1, 2, 3, \ldots\}$ = set of whole numbers.
$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ = the set of all integers

$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z}, q \neq 0\}$ = set of all rational numbers

$I = \{x : x \notin \mathbb{Q}\}$ = the set of all irrational numbers

$\mathbb{R} = \{\mathbb{Q} \cup I\}$ = the set of all real numbers.

"Relations in above sets"

$$\mathbb{N} \subset W \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}.$$
$$\cup$$
$$I = \mathbb{R} \setminus \mathbb{Q}$$

Note: Let $A = \{1, 2, 3\}$ then we can not say that $A \in A$
⇒ we will say $2 \in A$ or $3 \in A \ldots$

Subset: If every elements of a set A is also an elements of set B then A is called a subset of B and denoted by $A \subseteq B$.

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for example:

$A = \{1,2\}$, $B = \{3,1,2,4\}$ or $A = \{3,2,9\}$, $B = \{3,2,9\}$

$A \subseteq B$ $A \subseteq B$

Superset:- if A is a subset of B means, A is contained in B, we also say that B contains A or B is superset of A; or can be written as $B \supset A$.

Example, $A = \{1,2,3\}$, $B = \{1,2,4,5,3\}$

$B \supset A$.

equality of sets:- two sets A and B are equal if every element of A belongs to B and every element of B belongs to A mathematically we write $A = B$.

Example:- $A = \{1,2,3\}$, $B = \{1,2,3\}$

Note:- There is no set of all sets.

Sol:- let if possible, X be the set of all sets

i.e, $X = \{B \mid B \text{ is a set}\}$

By definition of set, X is a set

and by definition of X

$X \in X$ a $\Rightarrow \Leftarrow$.

Proper set:- If every elements of the set A is an elements of the set B and B contains at least one elements which does not belong to A

for example:

$A = \{3,4\}$ and $B = \{3,4,7\}$ then $A \subset B$.

Universal set:- we consider all the sets to be subsets of a given fixed set known as universal set it is denoted by U.

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* Finite set :- if a set consist of finite number of elements is called finite set.
  e.g :- $\{3, 7, 9\}$ is a finite set.

Infinite set :- if a set consist of an infinite number of elements, is called infinite set.
e.g. :- $\mathbb{Q}$, $\mathbb{Z}$, $\mathbb{R}$ etc.

Imp. power set :- The power set of set A denoted by $P(A)$ and defined by

$$P(A) = \{Y : Y \subseteq A\}$$. i.e, $P(A)$ is the

Collection of all possible subset of a set A. and $|P(A)| = 2^{|A|}$. It is never be empty i.e, $P(A) \neq \phi$.

NULL set :- A set consisting of no points is called the empty set or null set. It is denoted by ϕ or $\{ \}$.

Note :- power set of any set never be empty.

Singleton set :- A set consisting of a single elements is called a singleton set.
e.g. :- $\{7\}$, $\{a\}$ e.t.c.

pairwise disjoint set :- A family $\{A_n\}$ of sets is said to be pairwise disjoint if $A_t \cap A_s = \phi$ $\forall$ $t, s \in \Delta$ s.t. $t \neq s$.

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Set operations

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(1) <u>UNION</u>:- The union of two sets A and B. denoted by $A \cup B$ is defined as the set ~~{x : x ∈ A~~

$$A \cup B = \{ x : x \in A \text{ or } x \in B \}$$. It means

the set of points which belongs to one of the sets A and B, i.e which belongs to A or to B or to both.

(2) <u>Intersection</u>:- The intersection of two sets A and B denoted by $A \cap B$. is defined as the set

$$A \cap B = \{ x : x \in A \text{ and } x \in B \}$$. It means

the set of points which belongs to both A and B.

$$A = \{1, 2, 3\},\ B = \{1, 5, 7\},\ A \cap B = \{1\}$$

(3) <u>Complement of a set</u>:- The complement of a set A is denoted by A^c or A'. i.e. the set of all point in the universal set U which donot belongs to A. It can be defined as

$$A^c = U \setminus A = \{ x : x \in U \text{ and } x \notin A \}$$

(4) <u>Difference of sets</u>:- The difference A−B of two sets A and B is the set of points in A which do not belong to B.

i.e, $A - B = A \cap B^c$

(5) <u>Symmetric difference of sets</u>:-

The symmetric difference of two sets A & B is denoted by $A \Delta B$ and defined as

$$A \Delta B = (A - B) \cup (B - A).$$

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Important laws.

If A, B, C be any sets, then

(1) Commutative laws (i) $A \cup B = B \cup A$
(ii) $A \cap B = B \cap A$

(2) Associative laws (i) $(A \cup B) \cup C = A \cup (B \cup C)$
(ii) $(A \cap B) \cap C = A \cap (B \cap C)$

(3) Distributive laws
(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(4) De-Morgan's laws (i) $(A \cup B)' = A' \cap B'$
(ii) $(A \cap B)' = A' \cup B'$

(5) (i) $A - (B \cup C) = (A - B) \cap (A - C)$
(ii) $A - (B \cap C) = (A - B) \cup (A - C)$

modulus of a real numbers :-
The modulus (or absolute value or numerical value) of a real numbers x is denoted by $|x|$ and is defined by $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Results. non-negative

(i) $|x| \geq 0 \ \forall x$ and $|x| = 0$ iff $x = 0$
(ii) $|-x| = |x|$ (iii) $|x|^2 = x^2$
(iv) $|x+y| \leq |x| + |y|$
(v) $|x-y| \leq |x| + |y|$
(vi) $|x+y| \geq |x| - |y|$
(vii) $|x-y| \geq |x| - |y|$ (viii) $|xy| = |x||y|$
(ix) $|x| < K \Leftrightarrow -K < x < K$
(x) A subset 'S' of R is bounded iff there exists a real numbers $K > 0$ such that
$|y| \leq K \ \forall y \in S.$

* **Algebraic structure** :- A non-empty set with one or more operations defined on it is called an algebraic structure. If S is any non-empty set and * be an operation defined on S, then (S, *) is an algebraic structure.

Field :-
An algebraic structure (F, +, ·) is called a field if the following axioms are satisfied.
(1) Closure law of additions, $a+b \in F \ \forall a,b \in F$

(2) Commutative law of additions, $a+b = b+a \ \forall a,b \in F$
(3) Associative law of Additions
$$a+(b+c) = (a+b)+c \ \forall a,b,c \in F$$
(4) existence of additive identity :-
$\exists$ an element '0' in F such that
$$a+0 = a = 0+a \ \forall a \in F$$
This element '0' is called additive identity of F.
(5) existance of additive inverse :-
for $a \in F$, $\exists \ b \in F$ s.t. $a+b = 0 = b+a$.
This element b is called additive inverse of a

(6) closure law of multiplication, $ab \in F \ \forall a,b \in F$
(7) Commutative law of multiplication, $ab = ba \ \forall a,b \in F$.
(8) Associative law of " , $a(bc) = (ab)c \ \forall a,b,c \in F$
(9) Existence of multiplicative inverse
for any $a \in F$; $a \neq 0$ $\exists \ b \in F$ s.t.
$ab = 1 = b.a$; this element is called mult inverse of a
(10) Existence of multiplicative identity :-
$\exists$ an element $1 \in F$. such that $a.1 = a = 1.a \ \forall a \in F$
The element '1' is called multiplicative identity of a.